Numerical experiments with cooperating multiple quadratic snakes for road extraction

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Higher-order active contours or snakes show much promise for the extraction of complex objects from noisy imagery. These models provide an elegant mathematical framework for specifying the desired properties of target objects through energy functionals that can be minimized with standard optimization techniques. However, techniques to allow quadratic snakes to change topology during segmentation have not been fully exploited. Additionally, external forces for improving convergence of quadratic snakes have similarly yet to be explored. In this article, we propose a model that allows multiple quadratic snakes to split, merge, and disappear. Although the separate components of our approach have been introduced elsewhere by Cohen (1991), Xu and Prince (1997), and Rochery et al. (2006), this article is the first comprehensive empirical study of their performance on real-world complex network extraction tasks. We analyze the applicability of the model to road extraction from satellite images that vary in complexity from simple networks to large networks with multiple loops. We also analyze the effects of external forces enhanced by oriented filtering, gradient vector flow fields, and Canny edge detection. In a series of experiments, we found that the multiple cooperating quadratic snake model performs well on complex, noisy images. Our experiments also establish a performance improvement when the proposed quadratic model is coupled with the Canny-based gradient vector flow technique.

Key words: higher-order active contours; road extraction

1. Introduction

Extracting linear structures such as road, river, and blood vessel networks from images is a challenging problem with many practical applications. Although accurate methods exist for tracking (Geman and Jedynak 1996), it is very difficult to extract complete networks in the presence of noise and distracting features. Existing approaches to fully automatic road extraction include dynamic programming (Fischler et al. 1981, Barzohar and Cooper 1996), Markov random fields (Regazzoni et al. 1995, Tupin et al. 1998), and other techniques [see Auclair-Fortier et al. (2000) for a complete overview], but some of the most successful approaches for road segmentations (Fua and Leclerc 1990, Rochery et al. 2006), vessel segmentations (Tang and Acton 2004), and online video sequences (Sawano and Okada 2004), so far have been based on active contours or snakes.

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Classical active contour models (Kass et al. 1987, Cohen 1991, Cohen and Cohen 1993) provide an elegant framework for optimal estimation in image processing; rather than writing an algorithm to extract the object or region of interest, we simply consider an energy functional a minimum of which is achieved at a good solution. Then, given a new image, we use general optimization techniques to find a contour minimizing the energy functional.

The main drawback of the classical parametric active contour model is its lack of topological flexibility. When there are several objects in an image that need to be captured, we require manual initialization of multiple contours or active methods able to ‘break’ contours into multiple pieces (Samadani 1989, Durkovich et al. 1995, Ngoi and Jia 1999, McInerney and Terzopoulos 2000, Choi et al. 2001, Giraldi et al. 2003). Even if we allow contours to split and merge, we encounter the problem that individual snakes can intersect themselves and each other, requiring geometric constraints to prevent intersections (Ivins and Porrill 1995) or explicit detection and handling of intersections (Wong et al. 1998, Ngoi and Jia 1999, Ji and Yan 2002). This problem is especially troublesome when nested snakes are initialized inside one another.

So far, level set methods (Sethian 1999) appear to be the most elegant approach to the problems of active contour topological flexibility and self-intersection (Caselles et al. 1993, Malladi et al. 1995, Rochery et al. 2006). Level set methods, which represent a curve implicitly as the zero-level set of an evolving hypersurface, allow curves to automatically break or merge. However, level set methods do not readily admit imposition of arbitrary geometric constraints (McInerney and Terzopoulos 2000) or external forces, and they are relatively susceptible to image noise (Xu et al. 2000), so methods to obtain the topological flexibility of level set methods within the more mathematically flexible framework of explicit contour representations are still under active research (Delingette and Montagnat 2000, Li et al. 2005).

Our network extraction approach is based on Xu and Prince’s gradient vector flow (GVF, Xu, and Prince 1997), Rochery’s quadratic active contours (Rochery et al. 2006), and efficient algorithms for splitting, merging, and deleting contours presented in this article. The model can be easily modified to include other types of gradient vector flow models such as the generalized gradient vector flow (Xu and Prince 1998a), multidirection GVF (Tang 2009), etc. Our **quadratic multiple snake model** represents a compromise between the geometric snakes’ ability to split and merge easily and the parametric snakes’ flexibility to incorporate arbitrary constraints. We use quadratic constraints (Rochery et al. 2006) both to avoid self-intersections and loops and as a means to encourage capture of thin elongated objects such as roads, rivers, canal systems, pipes, and vascular systems. Our split and merge algorithms employ straightforward conditions on the closeness of nonadjacent contour points. In the model, separate snakes can repel each other but are still capable of approaching an object from opposite sides. The split and merge algorithms make it possible to extract highly complex networks of roads and other linear structures. The model thus provides the topological adaptability of geometric models without sacrificing the simplicity, efficiency, or flexibility of traditional parametric models.

In this article, we provide the mathematical motivation for the model and perform a series of numerical experiments to evaluate the effectiveness of the approach on real satellite data. We manipulate three factors:

- **Noise level**: We prepare noised images by adding varying levels of Gaussian noise to an original image containing a clear road network and evaluate the tolerance of the proposed approach against noise.

- **Type of external force**: We compare two types of the external force for contour evolution: one using Xu and Prince’s gradient vector flow (Xu and Prince 1997) applied
directly to the image gradient, and the one using a GVF applied to the gradient of a binary image resulting from oriented filtering combined with Canny edge detection.

- **Model order**: We contrast conventional linear snakes against quadratic snakes (Rochery et al. 2006) that allow contour points with opposite normals to repel each other unless they are aligned with opposite sides of the same ridge edge.

Although the separate components of our approach have been introduced elsewhere (Cohen 1991, Xu and Prince 1997, Rochery et al. 2006), this article is the first comprehensive empirical study of their performance on real-world complex network extraction tasks. For each active contour model and road network extraction task, we perform a detailed analysis of the model’s precision and convergence time. Our main finding is that the multiple cooperating quadratic snake model allowing splitting and merging of contours performs well on complex, noisy images with road networks of multiple scales.

2. Method

2.1. Quadratic snake model

This section provides a brief overview of the quadratic snake proposed by Rochery et al. (2006). An active contour or snake is parametrically defined as

\[ \gamma(p) = [x(p), y(p)]^T, \]  

(1)

where \( p \) is the curvilinear abscissa of the contour and the vector \([x(p), y(p)]^T\) defines the Cartesian coordinates of the point \( \gamma(p) \).

The energy functional is given by

\[ E_s(\gamma) = E_g(\gamma) + \lambda E_i(\gamma), \]  

(2)

where \( E_g(\gamma) \) is the geometric energy and \( E_i(\gamma) \) is the image energy of the contour \( \gamma \). \( \lambda \) is a free parameter determining the relative importance of the two terms.

To apply the method to road extraction, we define the geometric energy functional to be

\[ E_g(\gamma) = L(\gamma) + \alpha A(\gamma) - \frac{\beta}{2} \int \int \Psi(||\gamma(p) - \gamma(p')||) \, dpdp', \]  

(3)

where \( L(\gamma) \) is the Euclidean length of \( \gamma \), \( A(\gamma) \) is the area enclosed by \( \gamma \), \( \mathbf{t}(p) \) is the unit-length tangent to \( \gamma \) at point \( p \), and \( \Psi(z) \), given the distance \( z \) between two points on the contour, is used to weigh the interaction between those two points (Equation 4). \( \alpha \) and \( \beta \) are constants weighting the relative importance of the terms. The functional is a combination of two linear, Euclidean invariant terms: the area and the length. The length term acts as a regularizer, whereas the area term controls the expansion of the region. For positive \( \beta \), \( E_g(\gamma) \) is minimized by contours with short length and parallel tangents. If \( \alpha \) is positive, contours with small enclosed area are favored; if it is negative, contours with large enclosed area are favored.

The third quadratic (also Euclidean invariant) term is responsible for the interactions between points on the snake. The interaction function \( \Psi(\cdot) \) is a smooth function expressing the radius of the region in which parallel tangents should be encouraged and antiparallel tangents should be discouraged:
\[ \Psi(z) = \begin{cases} 
1 & \text{if } z < d - \epsilon, \\
0 & \text{if } z < d + \epsilon, \\
\frac{1}{2} \left( 1 - \frac{z - d}{\epsilon} - \frac{1}{\pi} \sin \pi \frac{z - d}{\epsilon} \right) & \text{otherwise.} 
\end{cases} \tag{4} \]

In application to road extraction, \( d \) is the expected road width and \( \epsilon \) expresses the expected variability in road width. During snake evolution, weighting by \( \Psi(z) \) in Equation (3) discourages two points with antiparallel tangents (the opposite sides of a putative road) from coming closer than distance \( d \) from each other.

The image energy functional \( E_i(\gamma) \) is defined as

\[
E_i(\gamma) = \int n(p) \cdot \nabla I(\gamma(p)) dp \\
- \int \int t(p) \cdot t(p') \nabla I(\gamma(p)) \cdot \nabla I(\gamma(p')) \Psi(\|\gamma(p) - \gamma(p')\|) dp dp', \tag{5} \]

where \( I(x,y) \) is a grayscale image and \( \nabla I(\gamma(p)) \) is the gradient of \( I \) evaluated at \( \gamma(p) \).

The first (linear) term favors antiparallel normal and gradient vectors, encouraging counterclockwise snakes to shrink around or clockwise snakes to expand to enclose dark regions surrounded by light roads. The second (quadratic) term favors nearby point pairs with two different configurations, one with parallel tangents and parallel gradients and the other with antiparallel tangents and antiparallel gradients.

To find a curve \( \gamma \) minimizing \( E_i(\gamma) \), one obtains the Euler equations using the calculus of variations. Introducing the gradient descent method and ignoring flow in the direction tangent to \( \gamma \), one obtains

\[
n(p) \cdot \frac{\partial \gamma}{\partial t} = -\kappa(p) - z - \lambda \nabla^2 I(\gamma(p)) \\
+ \beta \int r(\gamma(p), \gamma(p')) \cdot n(p') \Psi'(\|\gamma(p) - \gamma(p')\|) dp' \\
+ 2\lambda \int r(\gamma(p), \gamma(p')) \cdot n(p') (\nabla I(\gamma(p)) \cdot \nabla I(\gamma(p'))) \Psi'(\|\gamma(p) - \gamma(p')\|) dp' \\
+ 2\lambda \int \nabla I(\gamma(p')) \cdot (\nabla \nabla I(\gamma(p)) n(p')) \Psi'(\|\gamma(p) - \gamma(p')\|) dp'. \tag{6} \]

In the equation, \( \kappa(p) \) is the curvature of \( \gamma \) at \( \gamma(p) \) and \( \nabla^2 I(\gamma(p)) \) is the Laplacian of \( I \) evaluated at \( \gamma(p) \).

\[
\gamma(p) \gamma(p') \frac{\gamma(p) - \gamma(p')}{\|\gamma(p) - \gamma(p')\|} 
\]

is the unit vector pointing from point \( \gamma(p) \) towards \( \gamma(p') \). \( \nabla \nabla I(\gamma(p)) \) is the \( 2 \times 2 \) Hessian of \( I \) evaluated at \( \gamma(p) \). \( z, \beta, \) and \( \lambda \) are free parameters that need to be determined experimentally. \( d \) and \( \epsilon \) are specified \textit{a priori} according to the desired road width.

### 2.2. GVF external force

The term \( z \Psi(\gamma) \) in Equation (3) leads to the constant term \( -z \) in Equation (6). This term provides a force similar to the ‘balloon force’ introduced by Cohen and Cohen (1993). It
increases the capture region around objects, but its effect is uniform throughout the image. This makes it difficult to specify a value for \( \alpha \) that is appropriate in all regions of the image.

Xu and Prince (1997, 1998b) have proposed to use, rather than a global balloon force, a smooth, diffuse gradient field as a local external force with the traditional linear snake. They find that this technique, gradient vector flow (GVF), improves the traditional snake’s convergence to a minimum energy configuration.

We propose the use of GVF with quadratic road extraction snakes (see Figure 1).

2.2.1. GVF

The GVF is a vector field \( \mathbf{V}(x, y) = [u(x, y), v(x, y)]^T \) minimizing the energy functional

\[
E(\mathbf{V}) = \int \int \mu \left( \|u\|^2 + \|v\|^2 + \|\nabla \tilde{I}\|^2 \| \mathbf{V} - \nabla \tilde{I} \|^2 \right) dx dy,
\]

where \( \tilde{I} \) is a preprocessed version of image \( I \), typically an edge image of some kind. The first term inside the integral encourages a smooth vector field, whereas the second term encourages fidelity to \( \nabla \tilde{I} \). \( \mu \) is a free parameter controlling the relative importance of the two terms. Minimizing functional (7) leads to the Euler equation given by

\[
\mu \nabla^2 \mathbf{V} - (\mathbf{V} - \nabla \tilde{I}) \| \nabla \tilde{I} \|^2 = 0.
\]

Equation (8) is then solved numerically by iterations. Furthermore, replacing \( \mu \) in Equation (8) by two weighting functions depending on \( \nabla \tilde{I} \) to control the relative importance of the two terms \( \nabla^2 \mathbf{V} \) and \( (\mathbf{V} - \nabla \tilde{I}) \| \nabla \tilde{I} \|^2 \) leads to the so-called generalized version of the GVF (Xu and Prince 1998a).

We obtain \( \tilde{I} \) using oriented filtering and Canny edge detection combined with elongated Laplacian of Gaussian filters that emphasize road-like structures, deemphasize nonroad-like structures, and, to a certain extent, fill in short gaps where a road has low contrast with the background. The resulting binary Canny image only includes information about road-like edges that have survived sharpening by the oriented filters. The GVF field on top of the sharpened edge image points towards the road-like edges from a long distance, and, during
snake evolution, it pushes the snake in an appropriate direction. This speeds evolution and makes it easier to find suitable parameters to obtain fast convergence.

2.2.2. Oriented filtering

Using oriented filters for contour detection, contour completion, and restoration of edges corrupted by noise is a recurring idea in image processing and computer vision [see, e.g., Knutsson et al. (1983), Perona and Malik (1990), Freeman and Adelson (1991), Steger (1998), Konishi et al. (2003)]. The oriented filters most frequently used are 2D Gabor filters (Daugman 1985) and directional Laplacian of Gaussian (LoG) filters. Gabor filters are thought to be good models of the response of simple cells in primary visual cortex (Jones and Palmer 1987). When paired symmetric (even) and antisymmetric (odd) oriented filter responses are combined by summing their squares, they are thought to be good models of the response of complex cells in primary visual cortex (Heitger et al. 1992). Perona and Malik (1990) advocated these paired ‘energy filters’ for their ability to detect not only step edges but also ridge edges at specific scales.

The ability of Gabor filters and LoG filters to detect ridge edges makes them ideal for identifying roads in satellite imagery. Our oriented filtering method is the same as that of Rochery et al. (Rochery et al. 2006). We use the linear response of elongated LoG filters tuned to detect roads at 8 orientations then (for bright roads with dark surround) take the minimum response over the 8 orientations. An example is shown in Figure 2.

Our convolution and minimum response selection procedure responds well to long straight edges having the effect of emphasizing road-like gradients, deemphasizing non-road-like gradients, and, to a certain extent, filling in short gaps where a road has low contrast with the background.

2.2.3. Obtaining the GVF field

After oriented filtering, we obtain the Canny edge image $\tilde{I}$ from the edge-enhanced image obtained from oriented filtering. This is the input to the GVF relaxation procedure (Xu and Prince 1997). We precalculate $V$ before snake evolution begins, then, similar to Xu and Prince, during evolution, for each point $\gamma(p)$, we add the force $\lambda_{GVF} n(p) \cdot V(\gamma(p))$ directly to the updated Equation (6). $\lambda_{GVF}$ is a weight trading off the importance of the GVF force against the other forces in Equation (6).

Clearly, this encourages the snake to snap to the road edge contours, where ideally $\|V(\gamma(p))\| = 0$.

Our experimental results show that this approach, combining the advantages of the GVF’s extended capture range and the quadratic snake’s flexibility, improves the snake’s convergence to configurations that accurately segment road-like structures.

Figure 2. Oriented filtering procedure for enhancing road contrast. (a) Original image. (b) Laplacian–of–Gaussian filters at 8 orientations, with horizontal standard deviation 1.0 and vertical standard deviation 3.0. (c) Pixelwise convolution minimum over all 8 orientations.
2.3. Family of quadratic snakes

A single quadratic snake is unable to extract enclosed regions and multiple disconnected networks in an image. We address this limitation by introducing a family of cooperating snakes that are able to split, merge, and disappear as necessary.

In our formulation, due to the curvature term $\kappa(p)$ and the area constant $\alpha$ in Equation (6), specifying the points on $\gamma$ in a counterclockwise direction creates a shrinking snake and specifying the points on $\gamma$ in a clockwise direction creates a growing snake.

An enclosed region (loop or a grid cell) can be extracted effectively by initializing two snakes, one shrinking snake covering the whole road network and another growing snake inside the enclosed region.

2.3.1. Splitting a snake

We split a snake into two snakes whenever two of its arms are squeezed too close together, that is, when the distance between two snake points is less than $d^{\text{split}}$ and those two points are at least $k$ snake points from each other in both directions of traversal around the contour (see, e.g., Figure 3a). $d^{\text{split}}$ should be less than $2\eta$, where $\eta$ is the maximum step size.

Figure 3. Cooperating snakes. (a) A shrinking snake splits into two snakes and finally captures two distant objects. (b) Two merging snakes. (c) Two shrinking snakes one of which has been deleted after reaching a minimally allowed length.
2.3.2. Merging two snakes

The merging algorithm selects points having high curvature and merges two snakes when (1) two selected points are closer than a prescribed minimal merging distance $d_{\text{merge}}$, (2) the traversal direction (clockwise or counterclockwise) of the two snakes is the same, and (3) the tangents at the two high curvature points are nearly antiparallel. Figure 3b shows an example of two snakes merging each other. High curvature points are those with $\kappa_\gamma(p) > 0.6\kappa_{\gamma}^{\text{max}}$, where $\kappa_{\gamma}^{\text{max}}$ is the maximum curvature for any point on $\gamma$. When these conditions are satisfied, the two snakes are combined into a single snake by deleting the high curvature points and merging at the holes.

Limiting the merge decision to high curvature points ensures that merging only occurs if two snakes have semicircular tips of their arms facing each other. It might seem that merging at low curvature points should also be permitted. However, as already explained, snakes normally repel each other because of the quadratic term in the internal energy Equation (3). Consequently, low curvature segments can approach each other when high-gradient features allow the external energy to overcome the geometric energy. When this occurs for low curvature segments, the two snakes are most likely positioned on different sides of a road and merging should not be allowed. There are several other (rare) cases when snakes face each other at low curvature parts. However, they should not be merged in those cases either.

Considering only the high curvature points also saves computational costs. In particular, the merging procedure requires computation of the angle between tangents only for the selected points. The number of those points usually does not exceed 10% of the total number of points.

The conditions that the traversal direction of two snakes should be the same and that the tangents at the two high curvature points should be antiparallel reflect the fact that in our system, nested snakes form a tree structure. We initialize all the snakes at the first level with the same direction of traversal. The second level has the opposite direction of traversal and so on. When two snakes from the same level merge, we assign the resulting snake the same direction. Snakes from two consecutive levels do not merge. Growing and shrinking behavior is controlled by the area constant ($a$) and the weight on the geometric energy ($\beta$).

2.3.3. Deleting a snake

A snake is deleted if it has perimeter less than $L_{\text{delete}}$ (see Figure 3c).

2.4. Experimental design

We present four experiments aimed at evaluating the cooperating snake model for road extraction. In Experiment 1, we evaluate the robustness of the model against noise. Beginning with an image containing a clear road network, we progressively add Gaussian noise. The initialization is a single contour along the boundary of each image. In Experiment 2, we evaluate the model on an image containing a road network with many loops, intersections, and distracting structures, shown in Figure 6. This image requires multiple initial contours and user initialization to successfully segment the road network while ignoring the background region surrounded by the road loop. Furthermore, extraction of this road network requires a model capable of splitting at artifacts created by the noise. In Experiment 3, we evaluate the model on an image containing a road network with widely varying widths. Although varying the parameter $\epsilon$ accommodates modest width variations, we need a consistent approach for roads with significantly different scales. Therefore, to
extract the entire road network, we independently optimize parameter sets for large-width roads and small-width roads, then we perform extractions using the two different sets of parameters, and then we finally combine the extracted large-width roads and small-width roads. The initialization is the image boundary in the large-width case, and multiple contours along the outlines of roads in the small-width case. Finally, in Experiment 4, we evaluate the spacial scalability of the model on an image containing a large-scale complex road network. We show an initialization with multiple snakes in Figure 11b. The images were obtained using Google Earth software.

We manipulate three factors:

- **Noise level**: We add four levels of Gaussian noise (25, 20, 15, and 10 dB) to the original image (see Figures 4 and 5).
- **GVF type**: In the first condition, PlainGVF, the GVF is calculated from the result of oriented filtering on image I directly. In the second condition, CannyGVF, the GVF is the result of oriented filtering and Canny edge extraction as described in Section 2.2.
- **Model order**: In the first condition, Linear, we use the simple linear snake model obtained by eliminating the quadratic terms from Equations (3) and (5). In the second condition, Quadratic, the full interactions in Equations (3) and (5) are included.

In Experiment 1, we only manipulate the noise level and use the quadratic snakes with the Canny-based GVF. In Experiments 2, 3, and 4, we only manipulate the GVF type and model order. With two levels for each of 2 factors, we have four possible manipulations for each of these experiments.

For each manipulation, we hand-tune the free parameters to achieve the best possible results. In oriented filtering, we adjust $\sigma_x$ and $\sigma_y$, the standard deviations of the elongated Gaussian, until we obtain clear road pixels. Generally, setting $\sigma_x$ to the approximate road

![Figure 4](image-url)
width and $\sigma_y$ to 1.5–10 times larger than $\sigma_x$ yields reasonable filtering results. Parameters $\alpha$, $\beta$, $\lambda$, $\lambda_{GVF}$, $d$, and $\epsilon$ are related to the snake evolution model. Larger $\alpha$ facilitates the avoidance of noisy spots with high intensity; however, care should be taken not to overwhelm the contribution of the GVF weighted by $\lambda_{GVF}$. We adjust $\beta$ to be large enough to prevent self-intersections in case of quadratic snakes, or set it to 0 otherwise. $\lambda$ should be larger for linear snakes than for quadratic snakes. This forces the linear snake to snap to road edges. Finally, we adjust the road width $d$ and its permissible deviation $\epsilon$ until we achieve the best agreement with the ground truth for the test data.

Figure 5. Experiment 1 results for noised images. (a1–a3) SNR = 25 dB. (b1–b3) SNR = 20 dB. (c1–c3) SNR = 15 dB. (d1–d3) SNR = 10 dB. Indices 1, 2, and 3 represent an original image, an oriented-filtered image, and extracted road networks, respectively.
We terminate contour evolution whenever the energy $E_s(\gamma)$ fails to decrease for some number of iterations. We measure not only the number of iterations but also the actual computation time. This is because the computational complexities of quadratic snakes and linear snakes are essentially different; $O(N \cdot \log N)$ for quadratic snakes and $O(N)$ for linear snakes, where $N$ is the total number of active contour points. Note that we have achieved the $O(N \cdot \log N)$ performance for quadratic snakes by optimizing the neighborhood search of contour points.

To evaluate the results, we create ground truth images manually, and we use them to calculate the precision (the proportion of detected pixels that are road pixels according to the ground truth), the recall (the proportion of road pixels that are detected), and $F_1$ (the harmonic mean of the precision and the recall) for each solution. Along with these pixel-based measures, we introduce the Hausdorff distance, averaged over all contour points, to measure the geometric similarity between the ground truth and the extracted road networks.

3. Results

3.1. Extraction of road networks from noisy images

The setup of Experiment 1 is given in Table 1. We demonstrate the robustness of the proposed quadratic method with Canny-based GVF against a practical range of noise. We observe from Table 2 that the proposed model performs well up to the 15-dB noise level. Note that, even in the extreme case of 10-dB noise, the snake succeeds in extracting the gross structure of the original road network, shown in Figure 5 (d3).
3.2. Extraction of road networks with loops

The setup of Experiment 2 is given in Table 3. The results are shown in Figure 7 and Table 4. The results demonstrate the clear superiority of the quadratic model over the linear model. See Table 4. The quadratic interaction model performs much better than the linear model in every case, indicating the importance of the quadratic geometric energy term. The quadratic model is also much better at preserving the road network graph structure. This comes at the cost of a factor of 6 slowdown in runtime performance.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\lambda_{GVF}$</th>
<th>$d$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonnoised</td>
<td>2.9</td>
<td>5.4</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
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<tr>
<td>SNR 25 dB</td>
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<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
</tr>
<tr>
<td>SNR 20 dB</td>
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<td>0.6</td>
<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
</tr>
<tr>
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<td>0.6</td>
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<td>2.8</td>
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<tr>
<td>SNR 10 dB</td>
<td>2.9</td>
<td>6.3</td>
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<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
</tr>
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</table>

3.3. Extraction of road networks with different widths

The setup of Experiment 3 is given in Table 5. Examining the results in Figure 9 and Table 6, we see that our approach works well for all manipulations. The quadratic models have a clear advantage over the linear models in correctly capturing thin roads. The bright spots left after oriented filtering in Figure 8 b and c can be ignored due to the split and delete topological operations. Note that the accuracy of the large-width stage (Table 6) is evaluated against only the large-width part of the network. However, the small-width stage detects the small-and

Table 1. Parameters in Experiment 1.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
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<th>$\lambda_{GVF}$</th>
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<td>2.8</td>
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<tr>
<td>SNR 20 dB</td>
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<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
</tr>
<tr>
<td>SNR 15 dB</td>
<td>3.3</td>
<td>6.6</td>
<td>0.6</td>
<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
</tr>
<tr>
<td>SNR 10 dB</td>
<td>2.9</td>
<td>6.3</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>2.8</td>
<td>0.75</td>
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</table>

Table 2. Extraction results of Experiment 1.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Iterations</th>
<th>Time (s)</th>
<th>Precision</th>
<th>Recall</th>
<th>$F_1$</th>
<th>Averaged Hausdorff distance</th>
<th>Graph structure preserved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonnoised</td>
<td>500</td>
<td>15.3</td>
<td>0.90</td>
<td>0.88</td>
<td>0.89</td>
<td>0.68</td>
<td>Yes</td>
</tr>
<tr>
<td>SNR 25 dB</td>
<td>500</td>
<td>14.1</td>
<td>0.92</td>
<td>0.87</td>
<td>0.89</td>
<td>0.69</td>
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<tr>
<td>SNR 20 dB</td>
<td>700</td>
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<tr>
<td>SNR 15 dB</td>
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<td>19.9</td>
<td>0.88</td>
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<td>0.87</td>
<td>0.86</td>
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<tr>
<td>SNR 10 dB</td>
<td>600</td>
<td>11.6</td>
<td>0.93</td>
<td>0.70</td>
<td>0.80</td>
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Table 3. Parameters in Experiment 2.

<table>
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<tr>
<th>Condition</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\lambda_{GVF}$</th>
<th>$d$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CannyGVF–Quadratic</td>
<td>1.4</td>
<td>11.2</td>
<td>0.14</td>
<td>0.48</td>
<td>6.4</td>
<td>0.4</td>
<td>1.8</td>
<td>0.70</td>
</tr>
<tr>
<td>CannyGVF–Linear</td>
<td>1.4</td>
<td>11.2</td>
<td>0.06</td>
<td>0</td>
<td>60.0</td>
<td>0.4</td>
<td>1.8</td>
<td>0.70</td>
</tr>
<tr>
<td>PlainGVF–Quadratic</td>
<td>1.4</td>
<td>11.2</td>
<td>0.14</td>
<td>0.48</td>
<td>6.4</td>
<td>0.4</td>
<td>1.8</td>
<td>0.70</td>
</tr>
<tr>
<td>PlainGVF–Linear</td>
<td>1.4</td>
<td>11.2</td>
<td>0.06</td>
<td>0</td>
<td>60.0</td>
<td>0.4</td>
<td>1.8</td>
<td>0.70</td>
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Table 4. Extraction results of Experiment 2.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Iterations</th>
<th>Time (s)</th>
<th>Precision</th>
<th>Recall</th>
<th>$F_1$</th>
<th>Averaged Hausdorff distance</th>
<th>Graph structure preserved</th>
</tr>
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<tr>
<td>CannyGVF–Quadratic</td>
<td>760</td>
<td>93.8</td>
<td>0.87</td>
<td>0.60</td>
<td>0.71</td>
<td>1.48</td>
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<tr>
<td>CannyGVF–Linear</td>
<td>260</td>
<td>15.3</td>
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<td>0.58</td>
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<td>PlainGVF–Linear</td>
<td>280</td>
<td>12.0</td>
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<td>0.29</td>
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</table>

Figure 7. Experiment 2. (a) Ground truth. (b) Snake initialization. (c) Extraction with condition CannyGVF–Quadratic. (d) Extraction with condition CannyGVF–Linear. (e) Extraction with condition PlainGVF–Quadratic. (f) Extraction with condition PlainGVF–Linear.

Figure 8. Experiment 3. (a) Original image. (b) Oriented-filtered image for large width. (c) Oriented-filtered image for small width. (d) Canny-based GVF for large width. (e) Plain GVF for large width. (f) Canny-based GVF for small width. (g) Plain GVF for small width.
Table 5. Parameters in Experiment 3.

<table>
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<th>Condition</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\lambda_{GVF}$</th>
<th>$d$</th>
<th>$\epsilon$</th>
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<tr>
<td>CannyGVF–Quadratic–Large</td>
<td>3.9</td>
<td>9.8</td>
<td>0.4</td>
<td>0.3</td>
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<td>0.2</td>
<td>4.2</td>
<td>0.7</td>
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<td>9.8</td>
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<td>0</td>
<td>6.0</td>
<td>0.2</td>
<td>4.2</td>
<td>0.7</td>
</tr>
<tr>
<td>PlainGVF–Quadratic–Large</td>
<td>3.9</td>
<td>9.8</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
<td>0.2</td>
<td>4.2</td>
<td>0.7</td>
</tr>
<tr>
<td>PlainGVF–Linear–Large</td>
<td>3.9</td>
<td>9.8</td>
<td>0.1</td>
<td>0</td>
<td>6.0</td>
<td>0.2</td>
<td>4.2</td>
<td>0.7</td>
</tr>
<tr>
<td>CannyGVF–Quadratic–Small</td>
<td>0.8</td>
<td>4.4</td>
<td>0.05</td>
<td>0.9</td>
<td>20.0</td>
<td>0.6</td>
<td>2.0</td>
<td>0.25</td>
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<tr>
<td>CannyGVF–Linear–Small</td>
<td>0.8</td>
<td>4.4</td>
<td>0.05</td>
<td>0</td>
<td>40.0</td>
<td>0.6</td>
<td>2.0</td>
<td>0.25</td>
</tr>
<tr>
<td>PlainGVF–Quadratic–Small</td>
<td>0.8</td>
<td>4.4</td>
<td>0.05</td>
<td>0.9</td>
<td>20.0</td>
<td>0.6</td>
<td>2.0</td>
<td>0.25</td>
</tr>
<tr>
<td>PlainGVF–Linear–Small</td>
<td>0.8</td>
<td>4.4</td>
<td>0.05</td>
<td>0</td>
<td>40.0</td>
<td>0.6</td>
<td>2.0</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 9. Experiment 3. (a) Ground truth for large width. (b) Ground truth for small width. (c) Ground truth for combined networks. (d1–d4) Extractions for large width. (e1–e4) Extractions for small width. (f1–f4) Extractions for combined networks. Indices 1, 2, 3, and 4 represent conditions CannyGVF–Quadratic, CannyGVF–Linear, PlainGVF–Quadratic, and PlainGVF–Linear, respectively.

high-width roads leading (formally) to poor accuracy. For instance, the averaged Hausdorff distance in case of CannyGVF–Quadratic–Large is 0.69, whereas the accuracy of CannyGVF–Quadratic–Small is 2.23. However, merging the two results produces good accuracy. The Hausdorff error of CannyGVF–Quadratic–Combined is still 0.69.
3.4. Extraction of large-scale road networks

The setup of Experiment 4 is given in Table 7. Preprocessing steps are illustrated in Figure 10. The results are shown in Figure 11 and Table 8. The extracted road networks (Figure 11 c-f) show the quadratic model’s ability to capturing the detailed structure of a large-scale complex network. This can also be confirmed by comparing the Hausdorff distances reported in Table 8. Along the GVF-type dimension, the Canny-based GVF provides more precise information on road edges than does the standard GVF. The relatively low precision of the quadratic models reported in Table 8 is due to the increased complexity of the road network and the snakes’ failures to penetrate high-contrast image regions such as those containing buildings or small road loops. This large, complex road network consists of approximately 120 road segments and 70 intersections.

4. Conclusion

In this article, we have formulated a model for multiple interacting quadratic active contours and analyzed its performance in road network extraction on a test bed of satellite images, characterized by varying network complexity. The experiments demonstrate a clear advantage for multiple quadratic snakes, particularly in the case of complex road networks with multiple loops. In Experiment 1, the quadratic model is able to tolerate approximately 15 dB of noise. Over Experiments 2, 3, and 4, the accuracy of the quadratic models in terms of Hausdorff distance is 50–80% better than the accuracy of the linear

---

Table 6. Extraction results of Experiment 3. (\(\cdot\)) is the sum of large-width and small-width experiments.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Iterations</th>
<th>Time (s)</th>
<th>Precision</th>
<th>Recall</th>
<th>(F_1)</th>
<th>Averaged Hausdorff distance</th>
<th>Graph structure preserved</th>
</tr>
</thead>
<tbody>
<tr>
<td>CannyGVF–Quadratic–Large</td>
<td>420</td>
<td>5.4</td>
<td>0.97</td>
<td>0.84</td>
<td>0.90</td>
<td>0.69</td>
<td>Yes</td>
</tr>
<tr>
<td>CannyGVF–Linear–Large</td>
<td>320</td>
<td>3.8</td>
<td>0.98</td>
<td>0.80</td>
<td>0.88</td>
<td>0.84</td>
<td>Yes</td>
</tr>
<tr>
<td>PlainGVF–Quadratic–Large</td>
<td>460</td>
<td>6.0</td>
<td>0.96</td>
<td>0.87</td>
<td>0.91</td>
<td>0.65</td>
<td>Yes</td>
</tr>
<tr>
<td>PlainGVF–Linear–Large</td>
<td>280</td>
<td>3.6</td>
<td>0.99</td>
<td>0.77</td>
<td>0.87</td>
<td>0.93</td>
<td>Yes</td>
</tr>
<tr>
<td>CannyGVF–Quadratic–Small</td>
<td>40</td>
<td>2.7</td>
<td>0.62</td>
<td>0.75</td>
<td>0.68</td>
<td>2.23</td>
<td>Yes</td>
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<tr>
<td>CannyGVF–Linear–Small</td>
<td>20</td>
<td>1.5</td>
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<td>2.64</td>
<td>No</td>
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<tr>
<td>PlainGVF–Quadratic–Small</td>
<td>60</td>
<td>3.7</td>
<td>0.66</td>
<td>0.73</td>
<td>0.69</td>
<td>2.23</td>
<td>Yes</td>
</tr>
<tr>
<td>PlainGVF–Linear–Small</td>
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<td>1.5</td>
<td>0.51</td>
<td>0.38</td>
<td>0.43</td>
<td>3.97</td>
<td>No</td>
</tr>
<tr>
<td>CannyGVF–Quadratic–Combined</td>
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<td>(8.1)</td>
<td>0.89</td>
<td>0.78</td>
<td>0.83</td>
<td>0.69</td>
<td>Yes</td>
</tr>
<tr>
<td>CannyGVF–Linear–Combined</td>
<td>(340)</td>
<td>(5.3)</td>
<td>0.95</td>
<td>0.60</td>
<td>0.73</td>
<td>2.21</td>
<td>No</td>
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<tr>
<td>PlainGVF–Quadratic–Combined</td>
<td>(520)</td>
<td>(9.7)</td>
<td>0.91</td>
<td>0.78</td>
<td>0.84</td>
<td>0.75</td>
<td>Yes</td>
</tr>
<tr>
<td>PlainGVF–Linear–Combined</td>
<td>(300)</td>
<td>(5.1)</td>
<td>0.96</td>
<td>0.52</td>
<td>0.67</td>
<td>3.40</td>
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</table>

Table 7. Parameters in Experiment 4

<table>
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<tr>
<th>Condition</th>
<th>(\sigma_x)</th>
<th>(\sigma_y)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\lambda)</th>
<th>(\lambda_{GVF})</th>
<th>(d)</th>
<th>(\epsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CannyGVF–Quadratic</td>
<td>1.3</td>
<td>7.2</td>
<td>0.02</td>
<td>0.48</td>
<td>40.0</td>
<td>0.32</td>
<td>1.08</td>
<td>0.73</td>
</tr>
<tr>
<td>CannyGVF–Linear</td>
<td>1.3</td>
<td>7.2</td>
<td>0.01</td>
<td>0</td>
<td>80.0</td>
<td>0.32</td>
<td>1.08</td>
<td>0.73</td>
</tr>
<tr>
<td>PlainGVF–Quadratic</td>
<td>1.3</td>
<td>7.2</td>
<td>0.02</td>
<td>0.48</td>
<td>40.0</td>
<td>0.32</td>
<td>1.08</td>
<td>0.73</td>
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<tr>
<td>PlainGVF–Linear</td>
<td>1.3</td>
<td>7.2</td>
<td>0.01</td>
<td>0</td>
<td>80.0</td>
<td>0.32</td>
<td>1.08</td>
<td>0.73</td>
</tr>
</tbody>
</table>
In terms of $F_1$, the quadratic models are 10–40% better than the linear models. In Experiment 4, the linear snake’s performance is clearly unacceptable, whereas the quadratic snakes perform quite well.

Our experiments also establish a performance improvement when the proposed quadratic model is coupled with the Canny-based GVF technique under circumstances. In Experiments 2, 3, and 4, the Canny-based GVF approach is 10–25% better than the plain GVF approach in terms of Hausdorff although the $F_1$ measure itself does not indicate any significant performance improvement of the Canny-based GVF relative to the plain GVF.

Overall, in Experiments 2, 3, and 4, we find that the quadratic snake model is capable of preserving the geometric structure of the road network, whereas the linear snake frequently fails to do so.

Given that our results approach the level of performance required by the GIS industry, the proposed model may eventually make automated extraction of road networks from satellite imagery practical. Future work will focus on computational optimization and automatic initialization of contours.
Acknowledgments
We thank the anonymous referees of this article for very useful comments and attention to details. This research was sponsored by Thailand Research Fund grants BRG5380016 and MRG4780209.

Note
1. For dark roads on a light background, we simply negate the terms involving the image. In the rest of the article, we assume light roads on a dark background.
References


